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RESEARCH MEMORANDUM

A PRELIMINARY THEORETICAL STUDY OF AERODYNAMIC INSTABILITY OF A TWO-BLADE HELICOPTER ROTOR

By

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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RESEARCH MEMORANDUM

A PRELIMINARY THEORETICAL STUDY OF AERODYNAMIC INSTABILITY OF A TWO-BLADE HELICOPTER ROTOR

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SUMMARY

A theory has been developed in preliminary form which seems capable of predicting the aerodynamic instability phenomena of a two-blade see-saw-type helicopter rotor. In particular, the theory indicates the possibility of unstable vibrations even with the chordwise center of mass at or ahead of the 25-percent-chord position.

The stability condition for oscillatory motion is expressed in terms of a small number of composite parameters that are evaluated from the moments of inertia, angle settings, and aerodynamic parameters of a blade.

Computed stability results for different coning angle settings, center-of-mass positions, and control-system stiffnesses for one value of blade density and aspect ratio are presented in a chart.

It is found that, in addition to parameters analogous to those occurring in wing-flutter theory, the present theory contains a parameter that represents an unstabilizing effect due to the difference between the moments of inertia in flapping and in rotation.

INTRODUCTION

The present preliminary paper presents some numerical results of flutter calculations for a two-blade helicopter. The results are published in preliminary form in order to make them more quickly available for possible application to the study of the phenomenon of blade "weaving," which is apparently an aerodynamic instability and has been called weaving from the appearance of the wavy path traced by the blade tips. The general method, which is an extension of the method of reference (1), is to be treated in more detail in a later report. Derivations and discussions of the method have consequently been largely omitted in the present paper. Several effects are omitted which undoubtedly influence the quantitative results but are believed to be unessential in a preliminary study of trends.

METHOD OF ANALYSIS

The computations are intended to apply to a helicopter having two rigidly connected blades set at a coning angle and pitch setting with respect to each other. The combined blades are treated as a single rigid body having three degrees of freedom in rotation about a fixed point at the hub.

The air forces are obtained from a blade-element analysis using the wing flutter theory of reference 2 but using the theoretical steady state value for the slope of the lift curve instead of the complex function $F + iG$. The effect of a flexible control system is represented by a spring in the blade feathering degree of freedom.

In the air-force terms, the noncirculatory and the circulatory terms are considered separately. The noncirculatory terms are treated as apparent added mass and moment of inertia and combined with the actual blade mass to obtain resultant blade, center-of-mass and moment-of-inertia parameters.

The equations of motion are obtained by first writing the Euler equations of motion for each blade, considered as a rigid body rotating about a fixed point. The air-force terms are expressed in terms of the angular-velocity components that occur in Euler's equations.

The equations for two blades are then combined in a way to represent a single rigid body. The two blades are jointed at fixed angles with respect to each other and in such a way that the respective lines of center of mass intersect in a point at the hub. The coordinate axes of Euler's equations are then transformed to axes rotating uniformly with the mean blade motion. A spring-stiffness term is then inserted to represent the effect of the control system.

When the various physical parameters have been combined into a smaller number of composite parameters to be defined in the following section of the report, the equations of motion in matrix form become:

$$\left\{\begin{bmatrix} \frac{D}{\Omega} + H_D & I_A & 0 \\ -I_B & \frac{D}{\Omega} + H_B & 0 \\ 0 & 0 & \frac{D}{\Omega} + H_E \end{bmatrix} \begin{bmatrix} \frac{D}{\Omega} & -1 & 0 \\ 1 & \frac{D}{\Omega} & 0 \\ 0 & 0 & \frac{D}{\Omega} \end{bmatrix} + \begin{bmatrix} K_\theta' & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\} \begin{bmatrix} \theta \\ \beta \\ \gamma \end{bmatrix} = 0$$

where

θ, β, γ feathering, flapping, and lagging angles, respectively

$D \equiv \frac{d}{dt}$

Ω mean angular velocity of rotor

and the other symbols are defined in the section, Physical Parameters.

If solutions are assumed of the form

$$\theta = A_\theta e^{\lambda\Omega t}$$

$$\beta = A_\beta e^{\lambda\Omega t}$$

$$\gamma = A_\gamma e^{\lambda\Omega t}$$

where A_θ, A_β, A_γ are constants, the determinantal equation for obtaining the values of λ is

$$\begin{vmatrix} \lambda^2 + H_D\lambda + I_A + K_\theta' & -\lambda(1 - I_A) - H_D & 0 \\ \lambda(1 - I_B) + H_B & \lambda^2 + H_B\lambda + I_B & 0 \\ 0 & 0 & \lambda^2 + H_E\lambda \end{vmatrix} = 0$$

The equation shows that the γ degree of freedom is independent of the others and need not be considered in the stability computations.

If the determinantal equation is expanded, a quartic in λ is obtained of the form

$$\lambda^4 + a\lambda^3 + b\lambda^2 + c\lambda + d = 0 \tag{1}$$

where

$$a = H_B + H_D$$

$$b = I_B + H_BH_D + I_A + K_\theta' + (1 - I_A)(1 - I_B)$$

$$c = I_BH_D + (I_A + K_\theta')H_B + (1 - I_B)H_D + (1 - I_A)H_B$$

$$d = (I_A + K_\theta')I_B + H_BH_D$$

for which the important stability condition for oscillatory motion is

$$c^2 - abc + a^2d < 0$$

or

$$-H_BH_DK_\theta'^2 + (H_B + H_D)(1 - H_BH_D - I_AI_B)H_BK_\theta' - (H_B + H_D)^2(1 - I_B)K_\theta' < 0$$

This can be written

$$K_\theta' H_B H_D \left[-K_\theta' + \left(1 + \frac{H_D}{H_B}\right)\left(I_B \frac{1 - I_A}{\frac{H_D}{H_B}} - 1 + I_B - E_B^2\right)\right] < 0 \qquad (2)$$

The critical condition for divergence is recognized from equation (1) as $d = 0$; but no further discussion of this type of instability will be given in the present paper.

PHYSICAL PARAMETERS

General Case

The final stability conditions are expressed in terms of certain composite parameters that are obtained from other basic parameters as follows: Let A, B, C denote the principal moments of inertia of a single blade (in a vacuum) about radial, chordwise, and perpendicular axes, respectively. The apparent added mass effect changes B to $B(1 + \kappa)$, where κ is the ratio of the mass of a cylinder of air of a diameter equal to the chord of the blade to the mass of the blade, both taken for equal length along the span. A suitable average value applies to tapered blades.

The effect of apparent added mass on A and C has been neglected in the present computations.

The following parameters involve the circulatory air force expressions for a single blade:

$$\left.\begin{aligned}
H_1 &= -\int_0^R 2\pi\rho b^2 a r^2 \, dr \\
H_2 &= \int_0^R 2\pi\rho b r^3 \, dr \\
H_4 &= -\int_0^R 2\pi\rho b^3 a\left(\frac{1}{2} - a\right) r \, dr \\
H_5 &= \int_0^R 2\pi\rho b^2\left(\frac{1}{2} - a\right) r^2 \, dr
\end{aligned}\right\} \qquad (3)$$

where r is the spanwise coordinate along a blade, R is the tip radius, and ρ, b, a have the same meaning as in wing flutter theory (reference 2), namely:

ρ mass of air per unit of volume

b half chord of blade

a chordwise distance of elastic axis (assumed to coincide with center of mass) aft of midchord, divided by b. Leading edge is -1; trailing edge is 1.

For the two blades rigidly connected so as to have a coning angle β_o and collective pitch setting θ_o the corresponding parameters, with apparent mass included, are:

$$\left.\begin{aligned}
A' &= A \cos^2\beta_o + B(1 + \kappa) \sin^2\theta_o \sin^2\beta_o + C \cos^2\theta_o \sin^2\beta_o \\
B' &= B(1 + \kappa) \cos^2\theta_o + C \sin^2\theta_o \\
C' &= A \sin^2\beta_o + B(1 + \kappa) \sin^2\theta_o \cos^2\beta_o + C \cos^2\theta_o \cos^2\beta_o
\end{aligned}\right\} \qquad (4)$$

$$\left.\begin{aligned}
H_1' &= H_1 \cos\theta_o \cos\beta_o - H_2 \sin\theta_o \sin\beta_o \left(\cos 2\theta_o \cos\beta_o + 2\cos\theta_o \sin\beta_o\right) \\
&\qquad - H_4 \sin\theta_o \sin\beta_o \cos\beta_o + H_5 \sin^2\beta_o \\
H_2' &= H_2 \cos\theta_o \left(\cos 2\theta_o \cos\beta_o - 2\sin^2\theta_o \sin\beta_o\right) \\
H_4' &= -2H_1 \sin\theta_o \cos\theta_o \sin\theta_o \cos^2\beta_o + H_4 \cos\theta_o \cos\beta_o \cos^2 2\beta_o \\
H_5' &= -2H_2 \sin\theta_o \sin\beta_o \cos\beta_o + H_5 \cos 2\beta_o
\end{aligned}\right\} \quad (5)$$

then

$$\left.\begin{aligned}
I_A &= \frac{C' - B'}{A'} + \frac{H_1'}{A'} \\
I_B &= \frac{C' - A'}{B'} - \frac{H_5'}{B'} \\
H_B &= \frac{H_2'}{B'} \\
H_D &= \frac{H_4'}{A'} \\
K_\theta' &= \frac{K_\theta}{A'\Omega^2}
\end{aligned}\right\} \quad (6)$$

where K_θ is the spring constant in the feathering degree of freedom.

Special Cases

For a homogeneous rectangular blade the expressions in equations (3) become

$$\left.\begin{aligned}
H_1 &= -2a\kappa B \\
H_2 &= \frac{3}{2}\kappa\frac{R}{b}B \\
H_4 &= -3a\left(\frac{1}{2} - a\right)\kappa\frac{R}{b}A \\
H_5 &= 2\left(\frac{1}{2} - a\right)\kappa B
\end{aligned}\right\} \tag{7}$$

also

$$\frac{B}{A} = \frac{R^2}{b^2} \tag{8}$$

If the blade mass is distributed in the plane of the span and chord

$$A + B = C$$

If β_o^3 and other third-order angle terms and certain other combinations of small terms are neglected, equations (4) and (5) become

$$\left.\begin{aligned}
A' &= A + B\beta_o^2 \\
B' &= B(1 + \kappa) - \kappa B\theta_o^2 \\
C' &= C - B\beta_o^2 + B\kappa\theta_o^2 \\
H_1' &= H_1\left(1 - \frac{\theta_o^2 + \beta_o^2}{2}\right) - H_2\theta_o\beta_o - H_4\theta_o\beta_o + H_5\beta_o^2 \\
H_2' &= H_2\left(1 - \frac{5\theta_o^2 + \beta_o^2}{2}\right) \\
H_4' &= -2H_1\theta_o\beta_o + H_4\left(1 - \frac{5\beta_o^2 + \theta_o^2}{2}\right) \\
H_5' &= -2H_2\theta_o\beta_o + H_5\left(1 - 2\beta_o^2\right)
\end{aligned}\right\} \tag{9}$$

Combining equations (6), (7), (8), and (9) gives

$$\left.\begin{aligned}
I_A &= \frac{1}{1 + \frac{B}{A}\beta_o^2}\left(1 - \frac{B}{A}\beta_o^2 + \kappa\frac{B}{A}\left\{-(1 + 2a) + (2 + a)\theta_o^2 + (1 - a)\beta_o^2\right.\right. \\
&\qquad \left.\left. - \frac{3}{2}\theta_o\beta_o\frac{R}{b}\left[1 - 2a\left(\frac{1}{2} - a\right)\frac{A}{B}\right]\right\}\right) \\
I_B &= \frac{1 - 2\beta_o^2 - 2\left(\frac{1}{2} - a\right)\kappa + \kappa\left[3\frac{R}{b}\theta_o\beta_o + \theta_o^2 + 4\left(\frac{1}{2} - a\right)\beta_o^2\right]}{1 + \kappa - \kappa\theta^2} \\
H_B &= \frac{\frac{3}{2}\kappa\frac{R}{b}\left(1 - \frac{5\theta_o^2 + \beta_o^2}{2}\right)}{1 + \kappa - \kappa\theta_o^2} \\
H_D &= \frac{-a\kappa\frac{R}{b}\left[3\left(\frac{1}{2} - a\right)\left(1 - \frac{5\beta_o^2 + \theta_o^2}{2}\right) - 4\frac{R}{b}\theta_o\beta_o\right]}{1 + \frac{B}{A}\beta_o^2} \\
\frac{H_D}{H_B} &= \frac{-a\left[3\left(\frac{1}{2} - a\right)\left(1 - \frac{5\beta_o^2 + \theta_o^2}{2}\right) - 4\frac{R}{b}\theta_o\beta_o\right]\left(1 + \kappa - \kappa\theta_o^2\right)}{\left(1 + \frac{B}{A}\beta_o^2\right)\frac{3}{2}\left(1 - \frac{5\theta_o^2 + \beta_o^2}{2}\right)} \\
K_\theta' &= \frac{K_\theta}{A\Omega^2\left(1 + \frac{B}{A}\beta_o^2\right)}
\end{aligned}\right\} \quad (10)$$

STABILITY CHART

Stability conditions in terms of physical parameters can now be computed from equations (2) and (10). Figure 1 is a plot of $\Omega/\sqrt{K_\theta/A}$ against β_o for fixed values of the other parameters. The quantity $\sqrt{K_\theta/A}$ may be recognized as the natural frequency of the control system and blade in feathering motion at zero coning angle.

The following constant values were assumed in the calculations:

$$\frac{B}{A} = 1000$$

$$\frac{R}{b} = \sqrt{1000}$$

$$\kappa = 0.018$$

$$H_B = 0.806$$

$$I_B = 1$$

These values could be realized approximately, in a blade having the following characteristics:

Radius, feet	23.75
Chord, feet	1.5
Weight, pounds	180

The value $\theta = 0$ has, for simplicity, been chosen in this first calculation. An inspection of the equations indicates that positive pitch angles θ make the rotor more unstable. The effect of the chordwise center of mass upon the stability is indicated by the different curves. It is noted in particular that instability can occur even if the center of mass is ahead of the 25-percent-chord position.

DISCUSSION OF RESULTS

The present theory appears to check, in a general way, the observations that instability of a two-blade see-saw rotor with a coning angle can occur even with the center of mass at or ahead of the 25-percent-chord position.

The theory shows that instability depends strongly upon coning angle and to a lesser extent upon pitch setting. A study of the parameters employed in the analysis shows that I_A and H_D change over a wide range with changes of coning angle and pitch setting, whereas I_B and H_B are relatively insensitive to angle changes. It appears after some experience with this type of analysis that the parameters I_A and H_D are useful ones for designers to study in considering the effect of various design changes upon stability. Changes that increase the positive value of I_A improve the stability. Examples are forward movement of the chordwise center of mass, decrease of coning angle, and decrease of aspect ratio. The parameter H_D arises from a term representing aerodynamic damping in feathering oscillation. This type of term is frequently ignored in studies of rotor-blade dynamics, but it seems to be important for evaluating the effectiveness of increased control system stiffness in overcoming instability. If H_D is arbitrarily allowed to approach zero in the stability condition (2), this condition reduces to

$$I_B\left(1 - I_A\right) < 0$$

which does not contain K_θ'. It can be shown that the curves of figure 1 then become vertical straight lines and erroneously predict that stability is independent of rotor speed and control-system stiffness.

Condition (2) shows that if a rotor is stable for $K_\theta' = 0$ it will be stable for all values of K_θ'. These points, corresponding to $K_\theta' = 0$, therefore represent important sufficient conditions for stability. If K_θ' is allowed to approach zero in condition (2), the sufficient condition for stability becomes

$$I_B \frac{1 - I_A}{H_D/H_B} - 1 + I_B - H_B^2 < 0$$

The part of this expression that is sensitive to angle changes is the parameter

$$P = \frac{I - I_A}{H_D}$$

which can be expressed in the form

$$P = \frac{A' + B' - C' + H_1'}{H_4'}$$

For stability the parameter P should not be larger than a certain positive quantity determined mainly by H_B. Changes in a ship that decrease the positive magnitude of P will improve the stability. If P is evaluated in terms of angles from equations (10), it becomes, with certain small terms neglected

$$P \approx \frac{R}{b} \frac{(1 + 2a) + 2\frac{\beta_o^2}{\kappa} + \frac{3}{2}\theta_o\beta_o\frac{R}{b}}{-a\left[3\left(\frac{1}{2} - a\right) - 4\theta_o\beta_o\frac{R}{b}\right]}$$

HOW TO PREVENT INSTABILITY

Insofar as the present theory has included the correct degrees of freedom and types of forces to account for the phenomenon of instability it can be used to indicate the effect of proposed remedies, such as the increase of control-system stiffness, forward displacement of chordwise center of mass, decrease of coning angle, or decrease of aspect ratio.

The general result of the analysis is that a see-saw rotor with a coning angle is more unstable than an airplane wing having corresponding parameters. The additional unstabilizing effect is associated with the difference in moments of inertia in flapping and in rotation. A rotor is most stable when the moment of inertia in rotation C' is larger than the moment of inertia in flapping B'. The main effect of increased coning angle appears to be to decrease C' without making a compensating change in B'. Configurations that tend to confine the mass distribution to the plane of rotation are therefore desirable for stability. Configurations that tend to spread out the mass in the plane of flapping are undesirable for stability.

The effect of mechancial damping has not been explicitly treated, but it is expected to have a stabilizing effect by general analogy with wing flutter theory.

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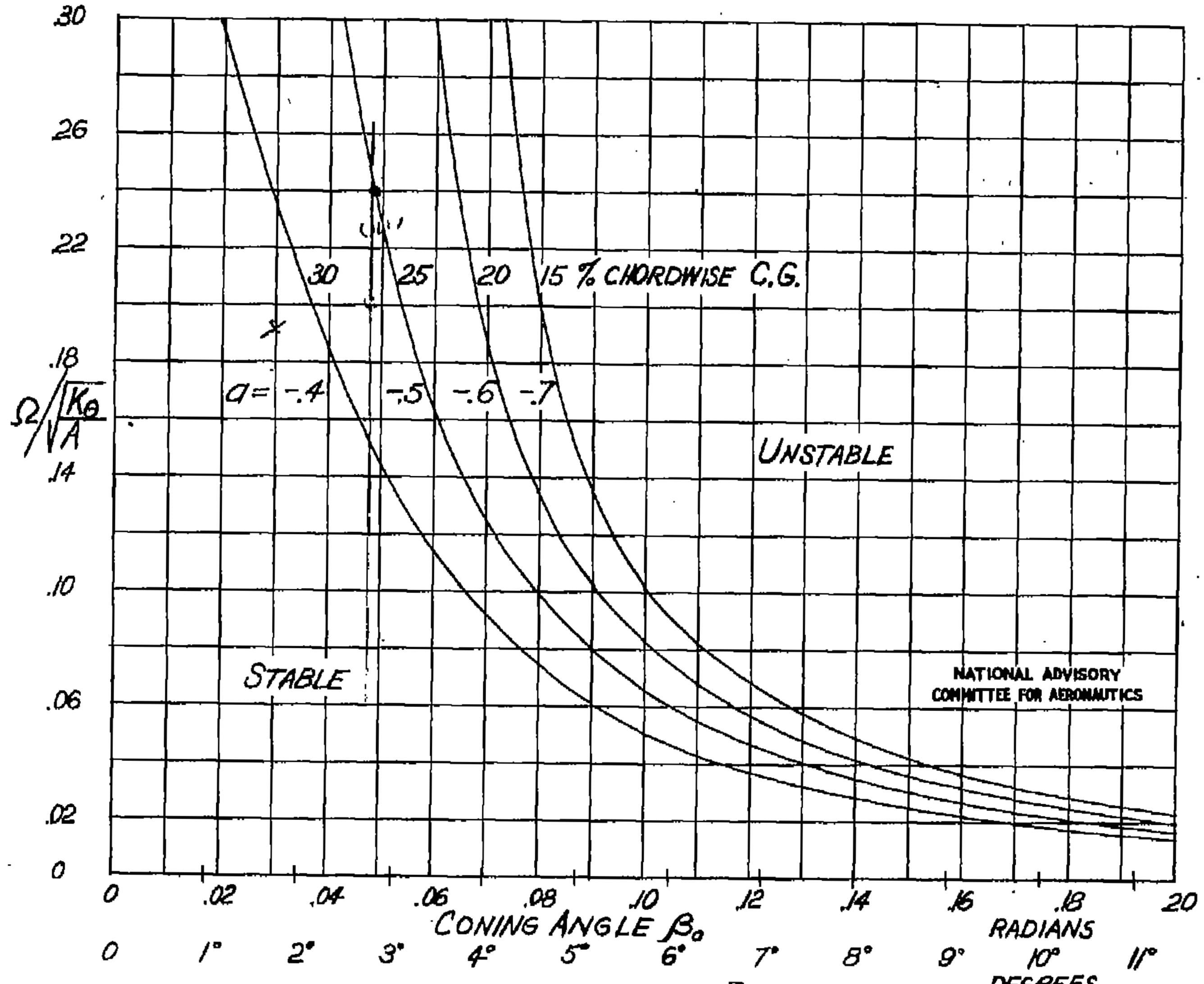

FIGURE 1.- STABILITY CHART FOR THE CONDITIONS $\frac{B}{A}=1000$, $\kappa=.018$, $I_B=1$, $\theta_o=0$.